# The rare traces of constructional procedures in "practical geometries"

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**Abstract**

From a sociological point of view, pre-Modern "non-theoretical geometry" is not adequately described as merely "practical". The "practical geometry" we find in written treatises is mostly that of "scribal" environments, and aims at *calculating* lengths, areas or volumes from already performed measurements. As a rule it is not interested in geometrical construction, nor in the making of measurements – the fields, broadly speaking, of master builders/architects and surveyors.

The paper discusses two cases – one fairly well-established, another more conjectural – where none the less "scribal" practical geometry does reveal traces of (very simple) geometrical construction. Both of these concern the "long run", connecting Old Babylonian, classical ancient and late medieval material. A final instance of weak communication between "scribal" and "surveying" geometry is located in thirteenth-fourteenth33-century France.

In a customary dichotomy, geometry (like many other fields, mathematical as well as non-mathematical) falls into "theoretical" and "practical". In full agreement with this, Stephen K. Victor [1979: ix] writes about his Ph.D.-project that

> My first assumption, and that of most of the people I have spoken to about the topic, was that "practical geometry" must relate somehow to architecture, surveying and city planning, to those areas, in other words, where geometry plays a central role in the exercise of other professions. The study of medieval buildings, fields and towns from extant physical evidence was not a fruitful approach for me, and I have left it to those better trained in the methods of archaeology and art history. Since I was working as a historian of science, I chose to concentrate on the written tradition of treatises called "practical geometry".

The treatises he chose to work on – the Latin late-twelfth–century *Artis cuiuslibet consummatio* and a vernacular (Picardian) *Pratike de geometrie* from the late thirteenth century which is largely a translation of the former work – led him to a different view,

> namely that practical geometry has its greatest importance as a popularization of mathematics. The treatises on practical geometry were a way of teaching some basic principles to those who would not remain in school or university long enough to become philosophers or theologians and would not necessarily exercise a mathematical profession, but who might want, or even need, some mathematics in their everyday lives. The sampling of arithmetic and astronomy in ACC and of commercial arithmetic and metrology in the Pratike argues for the generally pedagogic, rather than scholastic, purpose of the treatises. The development of a vernacular version of ACC is further evidence that the practical geometries sought their homes outside of the universities, perhaps in the bureaucratic and commercial milieus. Nonetheless, as the Introduction shows, the formalized structures of university education had an influence even on the non-scholastic tradition of practical geometry. As the tradition developed, the practical geometries acquired an increasingly theoretical underpinning, to the point where they are sometimes considered works on measurement rather than simply practical geometries.

Part of this conclusion depends critically on the choice of a Latin treatise and a vernacular treatise which in the main was derived from it. Other features, however, are shared not only with the Italian vernacular *Pratiche di geometria* and with Fibonacci's *Pratica geometrie* but also with most Arabic,[1] Sanskrit, Chinese, Greek, Babylonian and ancient Egyptian writings on the subject-matter. They deal – not at al or not much with *measurement*, as Victor states euphemistically, but rather with *how to calculate* something on the basis of measurements that have already been performed or which are presupposed to have been performed, either on pre-existing

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[1] An Arabic exception to this rule is Abū'l-Wafā''s *Book on What is Necessary for Artisans in Geometrical Construction* [ed., Russian trans. Krasnova 1966].

objects or on configurations which are supposed to have been already constructed.[2] In general terms, they belong within the "scribal" sphere, or in Victor's words, the "bureaucratic and commercial milieus".

None the less, a few traces of constructional procedures hide within these texts. I shall present two instances, one fairly certain and the other not much more than suggestive.

### Constructing the circular diameter

In *Metrica* I.xxx [ed. Schöne 1903: 74] Hero explains that "the ancients" – οἱ ἀρχαῖοι – in their formula for the area of a circular segment seem to have "followed those who took the perimeter to encompass the triple (τριπλάσιος) of the diameter", whereas I.xxxi [ed. Schöne 1903: 74] states that "those who made more precise investigations" must have followed the course according to which the perimeter is the triple diameter and in addition $\frac{1}{7}$ of the diameter.

Hero himself teaches (I.xxvi, ed. [Schöne 1903: 66]) to multiply the perimeter by 22 (using the construction "22 ἐπὶ") and then to take the seventh, but in the pseudo-Heronian *Geometrica* [ed. Heiberg 1912][3] – throughout using the "more precise" variant – the diameter is invariably taken "thrice" or "tripled", and this triple calculated explicitly, after which a supplementary seventh is added. The terms for tripling are invariably τρισσάκις or τριπλάσιον even when neighbouring multiplications are ἐπὶ *n*.[4]

The same terminological distinction between tripling and multiplication is found already in Old Babylonian geometry (c. 1700 BCE). Here, the perimeter is always found as the diameter "repeated until three" (*ana* 3 *eṣēpum*), or it is "tripled" (*šalāšum*) the diameter, not by the normal multiplication (*našûm*, "to raise") used, e.g., when the area of the circle is found as 5′ (= $\frac{1}{12}$) times the square on the perimeter.

The explanation for this linguistic puzzle is found in two texts from the fourteenth and the fifteenth century (CE). One is Mathes Roriczer's *Geometria deutsch* from c. 1488 [ed., trans. Shelby 1977: 121]:

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2 Actually, the genre studied by Victor – Latin practical geometries such as *Geometria incerti auctoris* and Hugh of Saint Victor's *Practica* – deals to some extent with mensuration, namely the determination of (e.g.) inaccessible heights by means of equilateral right triangles. This also had a slight (very slight!) impact on Italian abbacus geometries.

3 Definitely *not* Heronian, and actually a composite created by the modern editor from two rather incompatible manuscript groups, respectively A+C and S+V, as Heiberg [1914: xxi] points out.

4 Thus mss AC, **17.8**, between **17.7** and **17.9**, and ms. S, **17.6**, after **17.6** [ed. Heiberg 1912: 336, 334].

If anyone wishes to make a circular line straight, so that the straight line and the circular are the same length, then make three circles next to one another, and divide the first circle into seven equal parts,
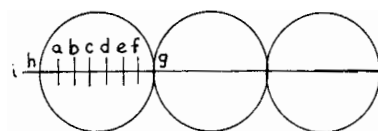


Figure 1. Roriczers construction of the circular perimeter as redrawn by Shelby [1977: 121].

one of which is marked out in continuation of the three circles – see Figure 1.[5]

The other is the old Icelandic manuscript A.M. 415 4to from the early fourteenth century, according to which (fol. 9ᵛ) "the measure around the circle is three times longer as its width, and a seventh of the fourth width",[6] obviously a reference to a similar construction.

Roriczer was a Gothic master builder; what he tells is the way to find by means of a drawing, *without calculation*, the perimeter corresponding to a given circular diameter. The Icelandic text confirms that the method was widespread; there seems to be little doubt that it offers the explanation why both the Greek and the Old Babylonian text refer to a tripling, a material repetition, and not to a mere numerical multiplication. This trick had thus been known for more than three thousand years in the late Middle Ages, first as a simple tripling, after the acceptance of the Archimedean improvement with an addition of a supplementary seventh – but still a separate supplement, and still to be provided in physical space.

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[5] Shelby [1977: 182] observes "some resemblamce between [Roriczer's procedure] and one of the theorems in a brief *Tractatus de quadratura circuli* – traditionally attributed to Camnpanus de Novara, but authorship and date uncertain". The passage in question [ed., trans. Clagett 1964: 591] deals with how to "give a straight line equal to a circularly drawn line", and runs as follows:

> Using mathematical knowledge and physical truth, a circle is divided into 22 equal parts, and with one part subtracted, that is, the 22nd part, a third of the remainder, namely, 7, is the diameter of the circle. Therefore, let the diameter be tripled and let there be added a seventh of the diameter, and let these parts be ordered in a straight line. We shall have a straight line equal to a circular line, as is apparent in the figure.

This could well be an attempted "theoretical" explanation of Roriczer's construction, but since the diagram shows a circle divided into 22 parts (with a diameter prolonged indefinitely toward the right) it could at least as well be a justification of the *calculation* found in the *Geometrica* and writings of the same kind, like that fifteenth-century *De inquisicione capacitatis figurarum* to which Shelby [1977: 6–65] refers in his introduction

[6] "Ummæling hrings hvers þrimr lutum lengri en bréidd hans ok sjaundungr of enni fiorðo breidd" [ed. Beckman & Kàlund 1914: 231f]. in [Menninger 1957: I, 91], which however gives no reference. I am grateful to Peter Springborg for localizing a passage which is quoted withour reference in [Menninger 1957: I, 91] and for providing me with a photocopy from the microfilm in the Arnamagnean collection, Copenhagen.

### *The regular octagon and the side-and-diagonal numbers*

The other example is differently balanced, in the sense that the traces in the calculational material are fewer but those in other sources more copious.

On trace is constituted by the Old Babylonian approximations to ration between the diagonal and the side of a square. One, already quite good, is 1;25 = $^{17}/_{12}$; the other, excellent, is 1;24,51,10. The former may have been found by iteration of a procedure also known from elsewhere in the Old Babylonian record, corresponding to the formula

$$\sqrt{n^2+d} = n+\frac{d}{2n}$$

– actually, the text VAT 6598 contains what may be a failed attempt at such iteration [Høyrup 2002: 271*f*]. The latter can be found by further iteration *by us*, but hardly by the Babylonians: as pointed out by David Fowler and Eleanor Robson [1998], the calculations have to pass through repeated divisions by very unpleasant sexagesimally irregular numbers; if we try to approximate by regular divisors (in agreement with what we know about Babylonian computational techniques), the reconstruction no longer yields the approximation it should but either one which is too rough or one which is even better.

Neugebauer and Sachs [1945: 43] propose a different way to the same approximations, namely through alternating arithmetical and harmonic means. Algebraically, this gives the same results – and computationally it runs into the same problems.

A third possibility – also algebraically equivalent – is the use of the "side-and-diagonal-number algorithm",

$$s_1 = d_1 = 1 , \quad s_{n+1} = s_n+d_n , \quad d_{n+1} = 2s_n+d_n .$$

The value of $2s^2-d^2$ oscillates between –1 (for odd *n*) and +1 (for even *n*). Since *s* and *d* increase, the ratio *d*:*s* therefore converges toward √2.

The procedure is first described by Theon of Smyrna (*Expositio* I.XXXI, ed. [Dupuis 1892: 70–74]), but according to his own statement in agreement with Pythagorean traditions without any addition whatsoever (book II, the introduction). It is also habitually assumed that Plato's reference to "a hundred numbers determined by the rational diameters of the pempad lacking one in each case" (*Republic* 546C, trans. [Shorey 1930: II, 247]) shows him to be familiar with the same algorithm. Actually, all it shows for certain is that he was familiar with the use of 7 as an (approximate) value for the diagonal in a square with side 5.[7] In any case, another discussion

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[7] Heath [1926: I, 399] and others read the "lacking one" as a reference to the fact that $7^2$ is lacking 1 compared to the square on the true (irrational) diameter in the square with side 5, which corresponds to an essential feature of the sequence of approximations produced

of the algorithm is found in Proclos's commentary to the passage in question from the *Republic*[8]. Finally, Proclos's commentary to *Elements* I contains an oblique but unmistakeable reference to the topic[9] and speaks of it as σύνεγγυζ, "proximate".

Though moderately to quite competent in mathematics, both Theon and Proclos have affinities to the environment which took mathematics as a way to or a kind of *gnosis* – in very loose terms, the Neopythagorean-Platonizing ambience. As I have discussed elsewhere [Høyrup 2001], this ambience, being unable to follow mathematics at the Euclidean or Archimedean level, borrowed its mathematics from the practitioners' level. Since no word about the algorithm has reached us from the ancient Greek high-level mathematicians, it seems reasonable to look for the roots of the procedure in some practitioners' environment.
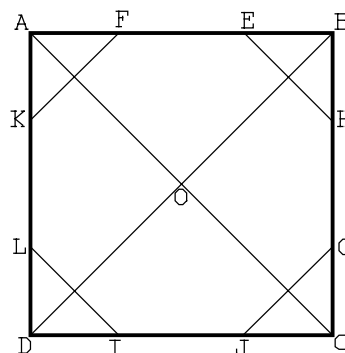


Figure 2. The construction of a regular octagon according to *De mensuris*.

The algorithm does not turn up as such in "mensuration" treatises, but the pseudo-Heronian *De mensuris* [ed. Heiberg 1914: 206] prescribes a construction of a regular octagon (under the misleading heading "mensuration of an octagon") which suggests the reasoning that may have led to its invention. In a square *ABCD*, the corners of the octagon *FEHGJILK* are found by making *AE = BF = BG = CH = ... = AO* – see Figure 2.

Figure 3 explains the correctness of the construction; the very same argument shows what we might call the "side-and-diagonal rule": namely that if *s* and *d* are the side and diagonal of a square, so will *s+d* and *2s+d* be.
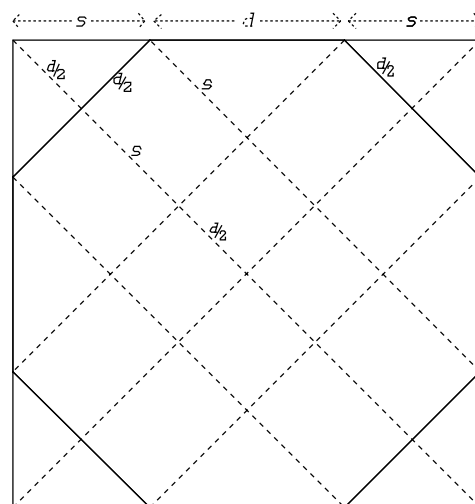
The same construction is found in several other sources: in Abū'l-Wafā''s *Book on What*



Figure 3. A diagram showing why the *De mensuris* construction works.

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by the algorithm. Actually, as pointed out to me by Marinus Taisbak (personal communication), Plato's point is rather that the number 48 (the number which is required) is lacking one with regard to the "number on the rational diameter 7" (and 2 with regard to that on the irrational diameter *dynamei*, as Plato goes on). This is indeed also Proclos's explanation, cf. Hultsch in [Kroll 1899: II, 407].

[8] Ed. [Kroll 1899: II, 24*f*]; cf. discussion in [Vitrac 1990: 351*f*].

[9] Ed. [Friedlein 1873: 427[21–23]], trans. [Morrow 1972: 339].

*is Necessary from Geometric Construction for the Artisan* as problem VII.xxii [ed., Russian trans. Krasnova 1966: 93]; in the *Geometria incerti auctoris* no. 55 [ed. Bubnov 1899: 360*f*]; in Roriczer's fifteenth-century *Geometria deutsch* [ed. Shelby 1977: 119*f*]; and in Serlio's *Primo libro di geometria* [1584: C2ʳ]. However, it is difficult to believe that anyone would get the idea to draw this diagram if the construction was not known already; and indeed, a much more intuitive diagram can be drawn, of which Figure 2 is simply a reduced version – namely the one shown in Figure 4. For symmetry reasons it is intuitively obvious that the superposition of two identical squares of which one is tilted 45° produces a regular octagon; but if we look at the diagram we also observe that $FR = RE = RP = KU$; this length we may call *s*; then the corresponding diagonal is $d = PF = AF = AK = RU$. Therefore, the semidiagonal $PO$ is $s+d+s = 2s+d$, thus equal to $AE$. Furthermore, since $KF = FE = 2s$, $UP = s+d$ and $KP = 2s+d$ are, respectively, the side and diagonal of a square – that is, the argument that shows the correctness of the *De mensuris* construction from this diagram also leads to the side-and-diagonal rule.

This construction was employed in actual architecture at least in Classical Antiquity: according to Hermann Kienast (personal communication) it can be seen to have been used in the ground plan of the Athenian "Tower of the Winds" from the first century BCE (outside the octagon itself, the point *P* is marked).[10] The superimposed squares producing the regular octagon are also found as an illustration to the determination of its area in Epaphroditus & Vitruvius Rufus [ed. Cantor 1875: 212, Fig. 40[11]]. Since the area is found from the octagonal number, this (as well as any) geometrical construction is irrelevant to the calculation; it can only be there because it was familiar. Finally, Roriczer's *Wimpergbüchlein* [ed. Shelby 1977: 108*f*] makes use of the configuration.[12]

The conclusions to be drawn from this are somewhat shaky. It appears that the construction of the octagon, both by means of superimposed squares and via the

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[10] The construction described by Vitruvius in *De architectura* I.vi.6–7 [ed., trans. Granger 1970: I, 58–61] is thus a (mistaken) reconstruction, explaining only how Vitruvius thought the construction *could* be made.

[11] The text is also in [Bubnov 1899: 539], but the diagram is omitted.

[12] Cantor [1907: 108] refers to the superimposed squares as common in Pharaonic wall painting, but this can hardly be considered as evidence, neither for use in actual architecture nor for mathematical reasoning based on it. But at least is shows the idea to be near at hand.

The several apparently regular octagons in Villard de Honnecourt's sketchbook [ed. Hahnloser 1935: Taf. 18, 63] are not accompanied by verbal or geometric indications as to how they were constructed. Only familiarity with Roriczer's description allows us to surmise that Villard's specimens were made in the same way; they cannot count as independent evidence.

simpler diagram of Figure 2, was known in Classical Antiquity and by late medieval Gothic masterbuilders; it is near at hand to assume some kind of continuity. In the absence of better explanation it is also tempting to presume that the side-and-diagonal algorism was inspired by one or the other of these constructions. Equally in the absence of better explanations, it is tempting to conjecture that the same algorism was used by Old Babylonian calculators, and that even they had come to know it in this way (nothing neither excluding nor guaranteeing that the Classical knowledge of the algorism was due to independent discovery).
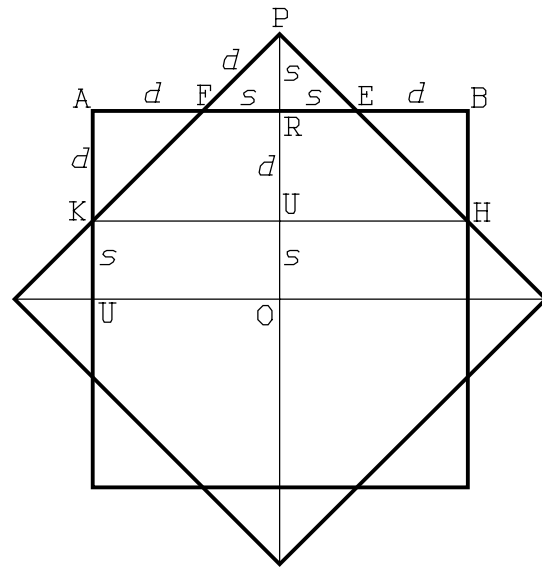
Figure 4. The completed version of Figure 2.

## Concluding observations

Fairly broad reading of writings on practical "mensuration" from a variety of pre-Modern cultures have thus permitted me to locate one rather certain instance of inspiration from a (very simple) *construction*, and one more dubious case. Even in this field it is confirmed that "practitioners' knowledge" was not unspecified "folk" but specialists' knowledge, and that specialists belonged to distinct cultures with little mutual communication.

As an illustration of the rarity of such communication I shall mention one instance, albeit rather of communication between "scribes" and surveyors than between "scribes" and constructors. In the introductory remarks I mentioned that the vernacular *Pratike de geometrie* was largely a translation of the Latin *Artis cuiuslibet consummatio.* However, on one point it is not (in fact on several points, but only this one concerns us here). The *Artis cuiuslibet consummatio* I.15 [ed. Victor 1979: 158–160] finds the area of an equilateral pentagon as the corresponding pentagonal number (in agreement with the agrimensorial tradition, and in spite of Gerbert's explanation of the fallacy in the triangular case [ed. Bubnov 1899: 45], even though this explanation is reported in chapter I.2 [ed. Victor 1979: 130]). In contrast, the *Pratike* [ed. Victor 1979: 489] suggests to multiply each side by half the height (which must be supposed to be measured, since no value is told) and to add the five partial areas afterwards.

A very similar procedure is proposed in the treatise *Geometrie due sunt partes principales* [ed. Hahn 1982:155], whose earliest manuscript also dates from the

thirteenth century. Here, for any regular polygon, it is proposed to construct the perpendicular bisector of each side, to see where they meet, and measure the heights – etc. Finally, the *Trattato di tutta l'arte dell'abacho*, written in 1334 in Tuscan language but in Montpellier and under obvious Provençal influence,[13] gives an alternative "by geometry" to a corrupt version of the "arithmetical" computation by means of the pentagonal number. This alternative looks as a mixture of the two Latin prescriptions – which can only mean that all three texts shared a common background, probably in French vernacular culture, where scribal "mensuration" had contact with real mensuration.

I know of no evidence beyond these three passages for the character of this point of contact, and it is much of an accident that I noticed them. Other evidence for interaction between different geometrical cultures of the time may be hidden in odd corners of manuscripts and wait for detection. On the other hand, the very possibility of hiding shows that such contacts were exceptions: on the whole, the pre-Modern geometrical cultures of scribal administrators, surveyors and master builders were as isolated from each other as, say, dentists, air traffic controllers and public relation experts nowadays.

## *References*

Beckman, N., & Kr. Kàlund (eds), 1914. *Alfræði íslenzk. Islandsk encycklopædisk Litteratur*. Vol. II. *Rímtǫl* København: Samfund til Udgivelse af Gammel Nordisk Litteratur / S. L. Møller, 1914–16.

Bubnov, Nicolaus (ed.), 1899. Gerberti postea Silvestri II papae *Opera mathematica* (972–1003). Berlin: Friedländer.

Cantor, Moritz, 1875. *Die römischen Agrimensoren und ihre Stellung in der Geschichte der Feldmesskunst. Eine historisch-mathematische Untersuchung*. Leipzig: Teubner.

Cantor, Moritz, 1907. *Vorlesungen über Geschichte der Mathematik*. Erster Band, *von den ältesten Zeiten bis zum Jahre 1200 n. Chr.* Dritte Auflage. Leipzig: Teubner.

Cassinet, Jean, 2001. "Une arithmétique toscane en 1334 en Avignon dans la cità des papes et de leurs banquiers florentins", pp. 105–128 *in Commerce et mathématiques du moyen âge à la renaissance, autour de la Méditerranée*. Actes du Colloque International du Centre International d'Histoire des Sciences Occitanes (Beaumont de Lomagne, 13–16 mai 1999). Toulouse: Éditions du C.I.H.S.O.

Clagett, Marshall, 1964. *Archimedes in the Middle Ages*. Vol. I. *The Arabo-Latin Tradition*. Madison, Wisconsin: University of Wisconsin Press.

Dupuis, J. (ed., trans.), 1892. Théon de Smyrne, philosophe platonicien, *Exposition des*

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[13] Florence, Biblioteca Nazionale Centrale, fond. prin. II,IX.57, fol. 133$^\text{v}$. For the date and place where the treatise was written, see [Cassinet 2001]. For the Provençal origin of much of the material, see [Høyrup, in progress].

*connaissances mathématiques utiles pour la lecture de Platon.* Traduite pour la première fois du grec en français. Paris: Hachette.

Fowler, David H., & Eleanor Robson, 1998. "Square Root Approximations in Old Babylonian Mathematics: YBC 7289 in Context". *Historia Mathematica* **25**, 366–378.

Friedlein, Gottfried (ed.), 1873. Procli Diadochi *In primum Euclidis Elementorum librum commentarii.* Leipzig: Teubner, 1873.

Granger, Frank (ed., trans.), 1970. Vitruvius, *De Architectura.* 2 vols. (Loeb Classical Library 251, 280). London: Heinemann / Cambridge, Mass.: Harvard University Press.

Hahn, Nan L. (ed), 1982. *Medieval Mensuration: QUADRANS VETUS and GEOMETRIE DUE SUNT PARTES PRINCIPALES ...* (Transactions of the American Philosophical Society, vol. 72, Part 8). Philadelphia: The American Philosophical Society.

Hahnloser, H. R. (ed.), 1935. *Villard de Honnecourt. Kritische Gesamtausgabe des Bauhüttenbuches ms. fr 19093 der Pariser Nationalbibliothek.* Wien: Anton Schroll.

Heath, Thomas L. (ed., trans.), 1926. *The Thirteen Books of Euclid's Elements*, Translated with Introduction and Commentary. 2nd revised edition. 3 vols. Cambridge: Cambridge University Press / New York: Macmillan.

Heiberg, J. L. (ed., trans.), 1912. Heronis *Definitiones* cum variis collectionibus. Heronis quae feruntur *Geometrica.* (Heronis Alexandrini Opera quae supersunt omnia, IV). Leipzig: Teubner.

Heiberg, J. L. (ed., trans.), 1914. Heronis quae feruntur *Stereometrica* et *De mensuris.* (Heronis Alexandrini Opera quae supersunt omnia, V). Leipzig: Teubner.

Høyrup, Jens, 2001. "Alchemy and Mathematics. Technical Knowledge Subservient to Ancient γνωσις", pp. 38–54 *in* Vincent F. Hendricks & Jesper Ryberg (eds), *Readings Philosophy and Science Studies*, vol. I. Roskilde: Department of Philosophy and Science Studies. Marred by numerous computer conversion errors.

Høyrup, Jens, 2002. *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin.* (Studies and Sources in the History of Mathematics and Physical Sciences). New York: Springer.

Høyrup, Jens, in progress. "Jacopo da Firenze, his *Tractatus algorismi* (1307) and the early phase of Italian abbacus culture" [Provisional title]. Preliminary book manuscript.

Krasnova, S. A. (ed., trans.), 1966. "Abu-l-Vafa al-Buzdžani, *Kniga o tom, čto neobxodimo remeslenniku iz geometričeskix postroenij*", pp. 42–140 *in* A. T. Grigor'jan & A. P. Juškevič, *Fiziko-matematičeskie nauki v stranax vostoka.* Sbornik statej i publikacij. Vypusk I (IV). Moskva: Izdatel'stvo »Nauka«.

Kroll, Wilhelm (ed.), 1899. Procli Diadochi *In Platonis Rem publicam commentarii.* 2 vols. Leipzig: Teubner, 1899, 1901.

Menninger, Karl, 1957. *Zahlwort und Ziffer. Eine Kulturgeschichte der Zahl.* I. *Zählreihe und Zahlsprache.* II. *Zahlschrift und Rechnen.* 2. neubearbeitete und erweiterte Auflage. Göttingen: Vandenhoeck & Ruprecht, 1957–58.

Morrow, Glenn R. (ed., trans.), 1970. Proclus, *A Commentary on the First Book of Euclid's Elements.* Translated with Introduction and Notes. Princeton, New Jersey: Princeton University Press.

Neugebauer, O., & A. Sachs, 1945. *Mathematical Cuneiform Texts.* (American Oriental Series,

vol. 29). New Haven, Connecticut: American Oriental Society.

Shelby, Lon R. (ed.), 1977. *Gothic Design Techniques. The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer.* Carbondale & Edwardsville: Southern Illinois University Press.

Schöne, Hermann (ed., trans.), 1903. Herons von Alexandria *Vermessungslehre* und *Dioptra. Griechisch und deutsch.* (Heronis Alexandrini Opera quae supersunt omnia, vol. III). Leipzig: Teubner.

Serlio, Sebastiano, 1584. *Tutte l'Opere d'Architettura.* Venezia: Francesco de' Francesci.

Shorey, Paul (ed., trans.), 1930. Plato, *The Republic.* 2 vols. (Loeb Classical Library 237, 276). London: Heinemann / Cambridge, Mass.: Harvard University Press, 1930, 1935..

Victor, Stephen K., 1979. *Practical Geometry in the Middle Ages. Artis cuiuslibet consummatio* and the *Pratike de geometrie.* Edited with Translations and Commentary. (Memoirs of the American Philosophical Society, vol. 134). Philadelphia; The American Philosophical Society.

Vitrac, Bernard (ed., trans.), 1990. Euclide d'Alexandrie, *Les Éléments.* Traduits du texte de Heiberg. Vol. I. Paris: Presses Universitaires de France.