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MATHEMATICS, ALGEBRA AND GEOMETRY

The Mathematical Context of the Bible

By JENS HØYRUP

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A. The traditions

It is not possible to speak of any specific *Biblical* mathematics. Neither the Old nor the New Testament were products of cultures carrying a mathematical tradition of their own above a normal level of »folk mathematics«. But both Testaments were products of cultures *in contact* with well-established and sophisticated mathematical traditions.

One of these is the Sumero-Babylonian tradition known from a wealth of cuneiform tablets. Though probably less vigorous than in the early second millennium B.C. it was still alive during the Babylonian exile; furthermore, it was apparently reflected in scribal education and in practitioners' ways throughout the Syrian area during the second and much of the first millennium B.C.

Another mathematical tradition of some importance for the Old Testament is that of Ancient Egypt. Already around the mid-second millennium B.C. was the Syrian orbit politically and commercially connected to Egypt; the Joseph story from *Genesis* 41 displays an Israelite view on precisely the features of Egyptian economy which molded Egyptian mathematics; and in the centuries of the Divided Monarchy the Egyptian hieratic number script was taken over.

The New Testament was written in the Hellenistic world and in Greek. In spite of this, the high, »theoretical«, level of Greek mathematics has left no traces in its text. Various quasi-philosophical currents dependent on Greek theoretical mathematics, however, are reflected both in the New Testament text and in Ancient and Medieval exegetical commentaries.

B. The »folk« substratum

The existence of a »native« ethnomathematical substratum among the early Hebrews follows mainly from indirect arguments: From all we know their cultural level was such that they needed it, as did all Near Eastern populations in the second and first millennia B.C. Hebrew *mēʾot*, »one hundred«, is also a common Semitic word. Hebrew *lʿʾōm*, »the people«, on the other hand, corresponds to Akaddian *lim*, »one thousand«, while Hebrew *ʾelep*, »one thousand«, corrresponds to Ethiopian «ten thousand«; linguistic reasons thus suggest that the counting range shared by early Semitic tribes did not exceed the hundreds.

A trace of »primitive« attitudes to numbers and counting can be found in *II Samuel* 24 (and *I Chronicles* 21), where David counts his people and is punished for his temerity; this fear (or taboo) of counting one's belongings is in fact widespread among populations who are either not familiar with or estranged from centralized states and administration.

Both because of the high numbers involved and because no traces of such estrangement from the ways of civilization turn up, the many other censuses found in the Old Testament can *not* be connected compellingly to the ethnomathematical substratum. The occurrence of borrowed Babylonian metrology (the *šeqel*) in textual vicinity of the important censuses in *Numbers* also speaks in favor of a possible borrowing of the habits and techniques of neighbouring older civilizations.

C. Babylonian mathematics

Babylonian mathematics was, in its origin, precisely an offspring of early »civilization« understood etymologically, as incipient *state formation*. Basically, it was a scribal activity, carried by scribes and similar practitioners and used for practical purposes—and since almost all practical applications of mathematics before the classical era consisted in *computation* of something, the unorthodox label »Babylonian computation« would fit the endeavour better than the name »mathematics« (which shall none the less be used in the following).

This does not mean that Babylonian mathematics consisted in nothing but a set of practitioners' recipes. Firstly, as it shall be argued below, Babylonian calculators knew what they were doing and why they did so. Secondly, like many professional environments making heavy use of mathematics, the Babylonian scribal culture produced a level of particularly complex, »pure« (i.e., not practically relevant) problems with appurtenant techniques, especially in the field of algebra (cf. below).

Traditionally, only the mathematics of the Old Babylonian and the Seleucid periods have been investigated and discussed in the literature. From the mid-1970es onwards, however, a number of texts have been discovered which permit to outline at least tentatively the development of Babylonian mathematics from the proto-Sumerian beginnings around 3000 B.C. to the Late Babylonian and Seleucid periods. Some of these texts and the conclusions drawn have been published but others (as of February 1989) only presented at the Workshops on Concept Development in Mathematics (Berlin(W) 1983, 1984, 1985, 1988), especially by Jöran Friberg, Peter Damerow, Robert Englund and Marvin Powell, Jr.

Already long before the late fourth millennium had a system of arithmetical recording or accounting based on small clay tokens been in use in the Near and Middle Eastern region (Schmandt-Besserat 1977). In the Uruk IV period (late fourth millennium, the period of state formation which also witnessed the development of writing), this system appears to have inspired both the development of writing and that of numerical and metrological notations. In as far as mathematics is concerned, furthermore, a trend toward harmonization of the various systems set in. So, the area unit sar (apparently meaning a »garden plot«, the area to be irrigated from a single well, and in any case a »natural unit«) came to be understood as the square of the basic length unit (the nindan, ≈6 m), and in general the whole system of area measures was keyed to the linear system (see Powell 1972); sub-unit metrologies were developed, as far as one can judge beyond the range of the traditional system; etc. The whole system was interconnected in a way which soon permitted coherent calculations linking arithmetically linear extensions, areas, time, and other quantities which belonged together in technical or social practice (part of the background for these statements has only been presented in workshops and is as yet unpublished; but see, e.g., some examples presented by Jöran Friberg (1984) and the implicit overview in Damerow & Englund 1987).

No doubt proto-Sumerian mathematics was created for the purposes of practical administration in what economic anthropology calls a »redistributive economy«; the replacement of »natural« but unconnected units by a complex of mathematically connected metrologies corresponds to the needs of the planning and accounting official rather than to those of the immediate producer. But the complexity of the system appears to go beyond even bureaucratic needs. Even though it is difficult to distinguish possible school tablets from indubitable administrative texts (only the latter contain officials' names) it is thus a fair assumption that the immediate root of the reorganization of a bundle of arithmetical techniques as coherent *mathematics* was the teaching in the temple school (this is argued more closely in Høyrup 1980: 14-17).

The early administration seems not to have distinguished bureaucratic from other priestly functions, and nothing in the mathematical substance distinguishes possible school exercises from other calculating texts. Only around the mid-third millennium is the term for »scribe« (dub-sar) found in the sources; at this time we also encounter non-bureaucratic use of the professional tools of the scribes: Literary texts and mathematical exercises beyond the context of daily administration, the latter dealing, e.g., with the division of extremely large numbers by »irregular« divisors like 7 and 33 (a theme which dominates the small group of mid-third millennium mathematical exercises from Šuruppak and Ebla—see Friberg (1986: 16-22) and Høyrup (1982). Even though such problems will have played no significant role in practical administration they were evidently a central concern for a scribal profession testing its own intellectual abilities.

The trend toward increasing regularization continued throughout the third millennium, and was brought to fruition in Ur III (21st century B.C.) (see Powell 1976). Early in the Ur III period an administrative reform was implemented which made extensive use of systematic and extremely meticulous book-keeping. It seems probable that it was for use in this context that the sexagesimal place value system was created (see NUMBERS AND COUNTING). Mathematical school exercises pointing beyond the administrative domain have not been found, and from parallels in other cultural domains it seems to be a reasonable assumption that the centralized state had drained the sources for scribal autonomy and thus for further development of non-utilitarian mathematics.

Non-utilitarian mathematics was, on the other hand, central to Old Babylonian mathematics, which is well documented in the sources (1900 to 1600 B.C., mainly the second part of this time-span). In this period, which was characterized by a highly individualized economy (compared to other Bronze Age cultures) and by an ideology emphasizing the individual as a private person, the scribal school developed a curriculum which stressed virtuosity beyond what was practically necessary; the triumphs of Babylonian »pure« mathematics, not least the »algebra«, appear to be a product of precisely this

Old Babylonian scribal school and scribal culture (see Høyrup 1985: 10-16).

Until Ur III, all mathematical texts had been in Sumerian; even in Semitic-speaking Ebla, Sumerian mathematics was taken over in the original language. Old Babylonian mathematics, on the contrary, was written in Akkadian—supplementary evidence that it represents a new *genre* and a break with the (plausibly more purely utilitarian) Ur III-tradition. Truly, quite a few texts are written predominantly by means of logograms of Sumerian descent; grammatical analysis shows, however, that all but a handful of these word signs are simply elliptic representations of Akkadian words and sentences.

Many mathematical tablets from the Old Babylonian period onwards are compilations, containing a variety of problems. Often, utilitarian and »pure« problems are found together; but mathematical and non-mathematical matters are not treated in the same texts. Obviously, Old Babylonian mathematics was not divided into fully distinct disciplines; on the other hand, mathematics *as a whole* was an autonomous concern—perhaps even (in the form of engineering, surveying and accounting or as a teacher's specialty) a distinct vocation.

In 1600 B.C., the Kassite conquest put an end to the Old Babylonian social order, to the age-old scribal school, to the characteristic Old Babylonian scribal ideology—and at the same occasion to the characteristic form of Old Babylonian mathematics. Scribal training was from now on taken care of by scribal »families« as apprenticeship; to a certain degree, mathematics came to be mixed up with other subjects on the same tablets, having lost its disciplinary autonomy; and the

»mathematician« would from now on identify himself in the colophons of tablets, e.g., as »exorcist« (*āšipu*) or »priest« (*šangû*).

In the first centuries after the Kassite conquest, mathematical texts are even virtually non-existent; a few Late Babylonian mathematical tablets have been discovered recently (one of them will appear in Friberg & Hunger, forthcoming). In the Seleucid era, the development of computational astronomy (starting already under the Achaemenids) gave rise to a renaissance of numerical computation and, as a sequel, of some of the old »pure« problems.

As already stated, Babylonian »mathematics« spells »computation«. In intermediate calculations, it made use of the sexagesimal place value system (see NUMBERS AND COUNTING). The use of this system, and the conversion of metrological values into »pure numbers« (and reversely, after a result was found) presupposed extensive use of mathematical, metrological, and technical tables. The first group encompasses tables of multiplication and of reciprocals (the division m/n was carried out as a multiplication $m \cdot {}^{1}/_{n}$); tables of squares and square roots, and of cubes and cube roots; of the root n of n^3+n^2; and even quite a few tables of successive powers of a number. The second group contain tabulated conversions of metrological values into sexagesimal multiples of the basic unit (corresponding to this list for classical English currency: »1 s. = 0.05 [*viz.*, £]; 2 s. = 0.1; 3 s. = 0.15; etc.); technical tables, finally, contain »fixed factors« to be used in technical computation (the ratio between the squared diameter and the area of a circle; the quantity of bricks to be carried by one worker over a given distance in one day; etc.).

The basic contents of Babylonian utilitarian mathematics correspond to these tables: Multiplication tables, tables of reciprocals and metrological tables were aids for calculation, and the technical tables constituted the nexus between mathematical computation and administrative and engineering reality. Mathematics was taught in school because the scribes should be able to calculate the areas of fields and the volume of canals to be dug out and siege ramps to be built and, not least, the manpower needed for that. All these calculations were made pretty much as they would be made today, with one important exception: The Babylonians had no concept of quantifiable angle and hence nothing similar to trigonometry. In practical mensuration, they would divide complicated fields into *practically right* triangles, *practically right* trapeziums and *practically rectangular* quadrangles (distinguishing, we might say, a *right* from a *wrong* angle); they would then calculate as we do, knowing that their results were not absolute truth but apparently without any definite idea about the nature and size of the errors. Presumably, they would see no decisive difference between the imprecision of manpower calculations and those of area determinations.

With these qualifications, the Babylonians knew the area of a right triangle (in practical mensuration, they would divide an obviously non-right triangle into two; in school exercises they might use the semi-product of the two »best« sides). In a Late Babylonian text we also find the calculation of a height (by means of the »Pythagorean theorem« known already in the Old Babylonian period). Similarly, they would find correctly the area of a rectangle and of a trapezium considered »right«. The area of an irregular quadrangle might be found by means of the »surveyors' formula«, as average length times average width. In practical mensuration, this technique has probably only been used for fairly regular quadrangles, where it gives acceptable results. In school texts it is also used as a pretext for formulating algebraic problems in cases where it is extremely unrealistic. The area of the circle was normally found as $^1/_{12}$ times the square of the circumference (corresponding to π=3), and the circumference as thrice the diameter. (One table of constants, however, has been assumed to contain a correction factor corresponding to $\pi=3^1/_8$).

Prismatic and cylindrical volumes were calculated as base times »height« (*viz.*, a side approximately perpendicular to the base). The volume of a truncated cone was found as that of a cylinder with the average diameter (which is correct for a cylinder, and only $^3/_4$ of the true value in the extreme case where the cone is not truncated), and that of a truncated pyramid in one text as height times average base (in another text perhaps correctly). When in doubt, once again, the Babylonians would opt for a (rather arbitrary) compromise instead of giving up in the face of theoretical difficulties.

Prismatic and cylindrical volumes were probably derived from a »naive« consideration of proportionality. The basic unit of area was the sar, and the basic unit of volume 1 sar times 1 cubit, also called a sar (to distinguish, modern historians speak of a »volume sar«). A prism with base *A* [sar] and height *1* [cubit] would then have a volume of *A* [volume sar]; if it were *h* cubits, whence *h* times as high, the volume would have to be $A \cdot h$. A corresponding argument of proportionality was apparently used when the height of a slope was found and in similar cases. Certain terminological considerations suggest that even the area of rectangular figures was originally thought of in this way.

A specifically Babylonian geometric problem-type is the partition of areas. Initially, this may have been a practical problem. No later than the 23d century B.C., however, it turns up as a »pure« problem: Which is the length of the transversal if a trapezium is bisected by a parallel transversal? In the Old Babylonian period even more complex problems of a similar kind are common, as are also a number of other more or less complex and more or less artificial division problems.

Many practical computations, of course, were not concerned with geometric entities but with quantities of grain to be levied as dues, with commercial exchange, etc. The techniques used can be illustrated by paraphrasing an illustrative problem: Two fields I and II are given, from one of which 4 gur (1 gur = 300 *qa*, 1 *qa* ≈ 1 liter) of grain are to be levied per bur (= 1800 sar), while the other yields a rent of 3 gur/bur. The total yield and the difference between the two areas is given. First everything is converted into sexagesimal multiples of the fundamental units sar and *qa*, in part through calculation, in part by means of a metrological table. The yield of that part of field I which exceeds field II is found. The remainder of the yield must them come from the remaining area *A*, which is composed from equal portions from field I and field II. The yield of one average sar is found, this is divided into the remaining yield, giving the remaining area, etc.

The idea behind the last step seems to be the »single false position« also known from other Babylonian texts: *If* the remaining area had been 1 *qa* it would consist of sar from each field, which permits that the yield be found as (say) *p qa*. In reality it is (say) $N \cdot p$ *qa*, and therefore the remaining area must be *N* sar.

The procedure gives an impression (confirmed by many other texts) of *ad hoc* improvisation, built on concrete thought rather than standardized techniques when we get beyond the most basic methods (conversions etc.). The same feature is also found in Old Babylonian second-degree and higher »algebra«, perhaps the most astonishing accomplishment of the Babylonian mathematical tradition. The term is put in quotes because it is not founded on symbols like post-Renaissance algebra, nor on words for unknown numbers as Medieval Islamic and Italian algebra. Instead, it builds on »naive« geometry: where modern algebra presents us with a problem $x^2+x=A$ (which may be transformed into $x \cdot (x+1)=A$), the Babylonians would consider a geometric rectangle whose length is known to exceed the width by *1*, and whose area is known to be *A*; where we transform the equation in order to isolate *x* the Babylonians would make corresponding cut-and-paste-transformations of the rectangle. The way they did it would be intuitively obvious, and they would provide no Euclidean proof that the procedure was correct (hence the term »naive«).

The basic transformations, e.g. the cutting up of rectangles, were made according to fixed schemes. But the Old Babylonian scribes would also solve quite complex problems, and when transforming them into simple problems they would make use of a stock of customary tricks but of no standard recipes—precisely as they did in arithmetical problems. When used with intelligence (as it is in many texts), Old Babylonian

»algebra« is therefore highly flexible: as long as we stick to one or two variables and to the second degree almost as flexible as (and in its sequence of operations very similar to) modern symbolic algebra. Only in more complex cases (from which the Babylonians did *not* abstain) do the disadvantages of their techniques become manifest.

Three qualifications should be given to the statement that Old Babylonian »algebra« was geometric. Firstly, the geometric entities involved were not abstract but concrete, measurable line segments and areas. Secondly, the geometric foundation did not prevent the technique from being applied to non-geometric quantities; as *we* represent, e.g., an unknown weight or an unknown price by a pure number x would the Babylonian represent them by a line segment of unknown (but numerically knowable) length. Naive-geometric »algebra« was an all-round way to find unknown quantities involved in complex relations. (Truly, only artificial relations. Babylonian scribal practice presented no problems of the second or higher degree; these had to be and were *constructed* in order to allow the display of scribal virtuosity).

Thirdly and finally, the statement is at cross-purposes with established beliefs. The interpretation which Neugebauer presented in the 1930es as a »first approximation« was at that time accepted at face value, and it has since then been conventional wisdom among historians of mathematics that Babylonian »algebra« was an algebra of numbers dealt with »rhetorically« as in the Arabic and Latin Middle Ages. Only recently has a detailed philological and comparative analysis of the text corpus and its terminology at large demonstrated that the numerical interpretation *is* in fact only a first approximation. (The reasons for this and the details of the reinterpretation are presented in Høyrup 1987).

A final important problem type is made up by numerical investigations. Some of these are connected to the computation of reciprocals, and hence to the needs of common computation. Others are inspired by the partition of the trapezium mentioned above, and lead to indeterminate problems for pairs or sets of numbers. The most famous of all such texts is the tablet is Plimpton 322, a table making use of sets of Pythagorean numbers (i.e., numbers a, b and c fulfilling the condition $a^2+b^2=c^2$).

Any mathematical corpus of knowledge is organized in a way which reflects its purposes, the ways of thought involved, and the underlying cognitive style. So was Babylonian mathematics as we know it. A general characteristic is its dominance by *methods*, not *problems*. At the first, utilitarian level this betrays that we know Babylonian mathematics from *school texts* which served to train future scribes in the methods of their profession. For that purpose, problems had to be constructed allowing the display of the methods to be trained. In practical life, on the other hand, the problems to be mastered were of course primary and the methods applied for that purpose secondary.

If we go to the »pure« level, however, we find the same primacy of methods, while Greek (and modern) pure mathematics takes *problems* as their starting point and develop the concepts and methods needed to surmount them. In this case, the training of practitioners explains nothing, since the particular methods belonging at this level had no practical application. Babylonian »pure« mathematics, however, had a purpose different from the scientific aim of Greek mathematics. As explained above, its rationale was the display of professional virtuosity; this is also the explanation why it flourished in the Old Babylonian era and disappeared from the archaeological horizon with the death of the scribal school.

Mathematical methods can be taught in two ways. One may present the methods in abstract terms, as theory, eventually to be illustrated by examples—or one may train them exclusively through paradigmatic examples. Nowadays, the former way is supposed to be used at higher educational levels, and the latter is reserved for the early stages of school. This was different in Babylonian mathematics, where we know of no case of formulated theory, and only of two or three where a paradigmatic example is used as the basis for sort of more general discussion of the method involved (though precisely these texts suggest that oral teaching would do so more often). The only case where rules are formulated in the abstract, is a couple of texts from Greek-ruled Uruk (Jöran Friberg, unpublished).

This feature of Babylonian mathematics can be compared to the make-up of Babylonian legal texts like the Codex Hammurapi. »Hammurapi's Law« is no law-book in the likeness of Roman Law. It is a collection of legal decisions made by the King, but of course only put together because the Royal decisions were supposed to serve as paradigms for the judges of the realm. We may also compare with the listing of hundreds of separate cases in Babylonian »omen science«.

One could say that Babylonian thought was more concrete and less inclined to abstraction than the modern mind. These terms, however, are used differently by a cognitive anthropologist like Levi-Strauss (1972) in his distinction between » savage« and modern mind. In other domains, Babylonian thought may be »concrete« in a Levi-Straussian sense, concrete entities acting as classifiers and imparting thereby some of their properties upon the class which they embody (as a primitive society may suppose the members of an »arrow clan« to be swifter than others). But already in the systematization of the omen literature is an underlying implicit abstraction visible in spite of its origin in magic thought (Larsen 1987), and at least Old Babylonian mathematics is still farther removed from Levi-Straussian concreteness. It was not their general mental make-up which prevented Old Babylonian scribes from transforming their (already autonomous) mathematics into abstract science, but rather a lack of motivation for doing so: The sort of »pure mathematics« which they created corresponded precisely to their socio-cultural needs, as the later development of Greek philosophy corresponded to that of the intellectually sophisticated stratum of the leisure class.

This is perhaps less true for the post-Kassite scribal priests, whose tablets might list together metrological conversions and the sacred numbers of gods (Friberg, personal communication concerning an unpublished tablet). Since early times, indeed, the technical cunning of scribes had been surrounded by sort of sacred aura. In the 22d century B.C. King Gudea of Lagash claimed that he had designed the plan of the temple, in the likeness of the scribal goddess »Nisaba, who knows the essence of counting«. From the mid-third millennium, »sacred numbers« were also associated with the gods, and numbers were used in writing according to the rebus-principle (the very first instance known integrates the two principles in a way reminding of Levi-Straussian »savageness«); the divine numbers went into the sacred calendar. In the early first millennium (before the development of mathematical astronomy), numbers were used cryptographically in a few astrological omen texts; in certain other texts, too, numbers were used for »coding«, in a way which may explain how the Assyrian King Sargon claimed the »number of his name« to be 16283. All these phenomena are hardly to be considered ingredients of Babylonian *mathematics*; but they reflect the existence and importance of mathematical activities, and do so most strongly in periods when mathematics was no autonomous endeavour (they are significantly absent from the sources for Old Babylonian scribal school mathematics); like marginal phenomena in general, they owe their existence to the core.

The main source collections for Old Babylonian and Seleucid mathematics have been published by Neugebauer (1935), Thureau-Dangin (1938), Neugebauer and Sachs (1945) and Bruins & Rutten (1961). The best overviews of the contents of Babylonian mathematics as known until 1975 are the ones by Vogel (1959; in German) and, especially, Vaiman (1961; in Russian, but a German translation is underway). A more popular introduction is due to van der Waerden (1962: 37-45, 62-81). An overview of the various interpretations of the Pythagorean triples of Plimpton 322 has been given by Friberg (1981). The first survey of third-millennium mathematics was published by Powell in 1976; recent discoveries of importance are presented by Damerow & Englund (1987); Englund (1988); Friberg and Hunger (forthcoming). A global overview including all recent discoveries is going to be published by Friberg (forthcoming), who has also written an excellent selective bibliography (in Dauben 1985: 37-51).

D. Syrian descendants

The Israelites will have encountered Babylonian mathematics during the exile, but only in the late phase when it was mixed up with Babylonian religion and divination and presumably already for that reason a suspicious subject. But long before that they will have been confronted with its descendants »at home«, in late second and early first millennium Syria.

After the mid-second millennium, the Canaanite city states of Syria were politically dominated by Egypt. Characteristically, however, the Canaanite kinglets and Pharao corresponded in Akkadian; Ugarit, the most prominent Canaanite state, developed its alphabetic script on the basis of cuneiform, and Ugaritic scribes were—just like their Hittite and Assyrian colleagues—taught according to the Sumero-Babylonian tradition (see Krecher 1969). The only traces of mathematics in their curriculum, however, consists in metrological lists. We may reasonable deduce that only the utilitarian stratum of Babylonian mathematics was adopted, while the »pure« superstructure was too much dependent on the particular Old Babylonian socio-cultural situation to be interesting in the Canaanite cultural outposts. The same will in all probability have been the case in early Israel, whose vista of Babylonian culture was much more indirect, somewhat later and (since it was largely mediated by the Canaanites) marked by distrust.

The closest point of contact will not have been the scribal but rather the ill-documented master builders' or architect's tradition. We are told in *I Kings* 5-7 and *II Chronicles* 2-3 that Solomon called in Phenician masters for the building of the Temple, and it seems indeed that they also followed Canaanite models (CAH II(2), 149). We have no direct testimony of the geometric lore of these masters; but an Islamic 9th century mensuration text by one Abū Bakr shows an astonishing degree of continuity with the Old Babylonian »algebra«, not only in mathematical substance and methods but down to the rhetorical and grammatical structure, and a story told by the late 10th century mathematician Abū'l-Wafāʾ suggests that the carriers of the continuous tradition be »artisans« (*sunnaʿ*), i.e., master builders and the like (see Høyrup 1986). There are even reasons to believe that the starting point for the Old Babylonian scribal »algebra«-tradition was a pre-existent artisans' tradition, although the evidence is not compelling, and the artisans may instead have been inspired by an originally scribal scheme.

Irrespective of its original relation to the Old Babylonian scribal tradition, the same artisans' tradition seems to have permeated then and later the whole Middle East; one Biblical reflection is well known: The »molten sea« set up by Solomon in the Temple is claimed (*1 Kings* 7: 23-24 and 2 *Chronicles* 4: 2) to posses a diameter of 10 cubits and a corresponding circumference of 30 cubits, corresponding to the above-mentioned »Babylonian π« of 3.

E. Egyptian mathematics

The other great Bronze Age mathematical tradition whose echoes can be traced in the Bible and, more distinctly, in the archeological remains of the Divided Kingdom, is that of Egypt. Though in many ways parallel to the Babylonian tradition, the two were obviously independent.

Like its counterpart, »Egyptian mathematics« is a scribal endeavour which should rather be labeled »computation«. It arose in connection with administrative needs in the early state; *Genesis* 41 provides an Israelite perspective on that particularity of Egyptian social life (compared to that of pre-Salomonic Israel) which called for extensive computation: Egyptian economy was, like that of the early Sumerian States, a *redistributive* system (the Biblical descriptions of Solomon's Temple building contain redistributive features, too). Correspondingly, the calculation of rations and of provision for workers are central topics in Egyptian mathematical texts, as are also the calculation of areas and of the volume of granaries.

It is not possible to distinguish a particular »pure« level in Egyptian mathematics. In that respect the two traditions differ. This is not to say, however, that Egyptian mathematics was a collection of recipes, nor (as we shall see below) that everything was always made in the way which suited practical applications best. There is, moreover, textual evidence that the scribes themselves saw their mathematical cunning as a high point of knowledge, as »rules for enquiring into nature, and for knowing all that exists, [every] mystery, ... every secret«—as Peet (1923: 33) translates the introductory passage of the Rhind Mathematical Papyrus (RMP in the following).

There are much fewer sources for the history of Egyptian mathematics than in the Babylonian case, and their chronological distribution is no less uneven. It is therefore only possible to give a very general overview of the historical development. The applicatiron of measures and the development of the metrological system began no later than the outgoing fourth millennium. Measures of capacity and of areas occur in texts from the third to fourth dynasties (c. 27th century B.C.). Already at the beginning of the first dynasty (late fourth millennium B.C.) the system of linear measures was used in the canon governing the pictorial representation of human beings (Iversen 1975: 60-66); and from an early date it must also have been used in architectural design.

No direct evidence for third millennium calculational techniques is available. From the way measurements and results are expressed, however, one can deduce that the later unit fraction system (see below) was not yet existent as a coherent system but only as a way to express *ad-hoc*-expansions of the systems of metrological subdivisions. We also know that the scribal calculators were taught as apprentices, in immediate practice, and not in a school (see Brunner 1957: 11-15).

All this was to change in the Middle Kingdom, at the beginning of the second millennium. Scribal education from now on took place in a school, and many texts are known which reflect the way professional self-esteem was inculcated in the future scribes. The introduction to the RMP quoted above shows that even mathematics served this purpose, just as in the Old Babylonian school.

A reorganization of the repertoir of fractions into a coherent unit fraction system appears to have taken place at this time. The old metrological subdivisions were conserved, but they were now supplemented by a systematic notation for abstract numerical fractions. Its basic elements were the unit fractions $^1/_2$, $^1/_3$, $^1/_4$, ..., $^1/_n$, ..., together with the complement $^2/_3$. Any fraction had to be expressed as a sum of such unit fractions in decreasing order (none of them identical). The Egyptian scribe would thus regard $^2/_5$ not as a number but as a problem, whose solution was $^1/_3+^1/_{15}$. For practical uses, these expressions were less handy than metrological subdivisions. For teaching purposes, however, they were better suited than subdivisions because everything could be expressed precisely; we may also assume that they played a role similar to that of Old Babylonian higher algebra, because the manipulation of unit factions required the same kind of mathematical virtuosity.

Once the unit fraction system had been introduced into the school curriculum, the scribes began using it in practical life too. At times the resulting contrast between gross, unnoticed errors and the meticulous precision of the notation may strike us; it is understandable, however, if we see the use of the system as sort of *art pour l'art*, as an expression of professional identity, and not as a merely utilitarian device.

Our main sources for the over-all contents and the techniques of Egyptian mathematics are two large papyri copied from Middle Kingdom originals, the Rhind Mathematical Papyrus (RMP) and the Moscow Mathematical Papyrus (MMP). The former is a fairly systematic handbook containing a wealth of intermediate calculations, while the latter is rather disorderly and apparently a students workbook. Especially the RMP is therefore excellent as a survey of Middle Kingdom Egyptian mathematics, and not very much is supplied by other sources beyond confirmation and clarification of dubious issues.

Almost a third of the RMP is devoted to the computation of $^2/_n$ (n=3, 5, ... 101) as a sum of unit fractions. This table is a prerequisite for all later calculations, because of the distinctive way in which the Egyptians performed multiplication and division: *Multiplying* a number A by 29 the scribe would find by successive doublings $2A$, $4A$, $8A$, and then $10A$ and, by another doubling, $20A$, and finally add A, $8A$ and $20A$ to find the result . That is, the whole procedure was founded on successive doublings and decuplings. If A contained fractions with an odd denominator, the doublings would involve use of the $2/n$-table; so, if $A={}^1/_5$, $2A={}^1/_3+{}^1/_{15}$, $4A={}^2/_3+{}^1/_{10}+{}^1/_{30}$, ... *Dividing* (as in RMP, problem 33) the number 37 by $B=1+{}^2/_3+{}^1/_2+{}^1/_7$, the scribe would calculate successively $2B$, $4B$, $8B$ and $16B$, seeing that $16B$ fills out 37 apart from a remainder which is 2 two times an implicit sub-unit $^1/_{42}$; since B is 97 times this same sub-unit, the remainder is twice $^1/_{97}B$, and the full result of the division is $16+{}^2/_{97}$, i.e., in the required system, $16+{}^1/_{56}+{}^1/_{679}+{}^1/_{776}$.

Simple multiplications and divisions might give the impression that Egyptian mathematics was purely additive. As shown, however, by the latter part of the division, as by many problem solutions making use of »false positions« (cf. above) and of free manipulation of appropriate sub-units, the Egyptian scribes had a perfect though implicit grasp of multiplicative relations and proportionality. Otherwise, indeed, they would have been unable to take care of their practical tasks.

A substantial part of the RMP aims at training the solution of problems arising through the use of the unit fraction system, especially in connection with problems of division and proportionality. Some such problems deal with abstract numbers, others make the connection to daily practice clear, e.g., when loaves are distributed to workers and foremen (with double rations for the latter), when the connection between unit fractions and various metrological systems are dealt with, or when the quality of beer and the size of loaves come in.

Another dominant interest is in geometrical computation. As in Mesopotamia, area measures are mathematically connected to linear measures, but even more clearly conceptualized as the product of a fixed standard width and a variable length. As in Babylonia, the concept of a quantifiable angle is absent, and triangular areas were found as the product of two sides containing a »practically right« angle. Trapeziums and trapezoids are absent from the sources, but the area of a circle is found as the square on D-$^1/_9D$ (D being the diameter), corresponding to $\pi = {}^{256}/_{81} = 3.16...$—much better than the normal Babylonian rule.

Prismatic and cylindrical volumes were of course found without difficulty; it is more astonishing that the volume of a truncated pyramid was found correctly (MMP, Problem 14). It is disputed whether a »basket« in MMP (Problem 10) is meant to be a hemisphere. If it is, its surface is found accurately (given the above-mentioned »π«); if a hemisphere is *not* meant, the computation suggests that the Egyptians would find the circular circumference (correctly) as the quadruple area of the circle divided by the diameter, and the area of semicylinder as the product of the curved and the straight side.

Geometry and geometrical computations were also used in Egyptian architecture. Architectural and building problems, however, are not very conspicuous in the mathematical texts, which in fact only contain two types: Firstly the calculation of the slope of pyramids; according to the RMP, where it is dealt with five times, this must have been a prominent problem type. Secondly the volume of a truncated pyramid, which is only known from MMP.

It is a recurrent claim that the Egyptians knew the Pythagorean theorem and used it in architectural construction. It should be observed, however, that the claim is not supported by any positive evidence. Many buildings, it is true, contain rectangles whose sides are to each other as 3 to 4; but nothing suggests that the Egyptians knew or were interested in the length of the diagonal.

Related to the use of geometry in architecture is the use in the pictorial arts of square grids and fixed proportions linked to the system of linear measures. This »canonical system« is one of the main factors creating the unique tenor of Egyptian art and upholding its stable character for several millennia, until a metrological reform in the 7th century B.C. made it instead a factor of change.

Anything similar to Babylonian second-degree algebra was absent from Egyptian mathematics. The closest we come are two types of geometric problems. One is found repeatedly in the MMP: In a (»practically«) right triangle, the area and the ratio between the sides containing the right angle are given; this is solved by means of a consideration of proportionality. The other comes from the Berlin Papyrus 6619 and can be translated into modern symbols as $x^2+y^2=100$, $y=\frac{3}{4}\cdot x$; the solution is obtained by means of a false position (»always take a square of side 1; then the other is $\frac{1}{2}+\frac{1}{4}$«).

These problems are atypical by being of the second degree. In fact, everything else related to algebra is of the first degree. But the techniques made use of are typical also of those first-degree problems which we would be tempted to solve algebraically. The »false position«, in particular, may be regarded as a »poor man's *x*«. The point in using an *x* is, in fact, that you can manipulate with the unknown quantity as if it were a known number; taking preliminarily the unknown to be *1* (or any other convenient number) gives you the same possibility, as long as you stick to »homogeneous problems« (i.e., problems which can be reduced to the type $x^n=A$).

The above description does not exhaust the contents of Egyptian mathematics, but it covers the principal features as far as we know them, and does so until the Assyrian domination. Then (modest) change set in: a number of Demotic mathematical papyri from the Hellenistic and Roman periods show, indeed, that material from the Babylonian or Middle Eastern practitioners' tradition had diffused into Egypt during the first millennium B.C. (perhaps carried by Persian military or fiscal surveyors?). Most conspicuous is an adoption of the

»Babylonian π«.

Egyptian mathematical texts are problem collections, just like the Babylonian ones. The closest we come to general descriptions of methods is a phrase like the »always take a square of side 1« quoted above. But even in Egypt the problems were meant to be paradigmatic—as told in the introduction to the RMP they were regarded as »rules«. *In the texts*, methods are thus primary and the problems secondary. In scribal professional practice, of course, the problems of real practice were primary, and since no clearly distinguishable level of non-utilitarian calculation developed in Egypt, the problems found in the texts are either real-life-problems or structurally similar to problems encountered in »real life«—including problems arising from the idiosyncratic multiplication and division algorithms and the unit fraction system. Globally regarded, the structure of Egyptian mathematics was thus determined by its practical duties and its characteristic methods and techniques, in mutual interaction and on an equal footing.

As in Babylonia, the mode of thought expressed in the mathematical texts is concrete. There is, however, one important difference. Babylonian mathematics, as we have seen, tended to represent other unknown entities by measurable geometric entities; the Egyptians, on the other hand, tended to represent everything by *pure numbers* (at least from the Middle Kingdom onward). Even though Babylonian mathematics is much more sophisticated in content than its Egyptian counterpart, the latter can thus be claimed to have gone farther in mathematical abstraction.

Old Babylonian mathematics, as we saw, appears to be purely secular. In later times, on the other hand, the borderline between mathematics, divination and religion seemed to be somewhat blurred. In Egypt, too, numeration and numbers played a religious-mystical role; in the *Book of Dead*, the deceased king is required to count his fingers (Neugebauer 1969: 9). But in spite of the »mysteries« and »secrets« spoken of in the RMP-introduction, the mathematical texts themselves appear to be devoid of religious and occult connotations.

This is of course in disagreement with the wide-spread speculations on »pyramid mysticism«. The pyramidological arguments build (at best) on a variety of numerical ratios purportedly found in the Cheops pyramid and claimed to reflect a precise knowledge of π and the »golden section«. Two flaws, however, characterize these assertions (see Robins & Shute 1985). Firstly, precise measurement of the (original!) dimensions of the worn-down pyramid is difficult, and in order to obtain their favorite ratios the pyramidologists avoid using the best measurements. Secondly, nothing in the mathematical texts suggests the slightest interest in the numbers claimed to be embodied in the pyramids; so, e.g., the Egyptians did not use a number corresponding to π but instead an approximation (*viz.* $^8/_9$) to $\sqrt{(\pi/4)}$, which is quite another entity though of course just as mathematically serviceable. On the other hand, the best measurements of pyramidal slopes correspond precisely to the way pyramidal slopes are indicated in the RMP, and come out mostly as 5 palms 1 finger or 5 palms 2 fingers horizontally per vertical cubit (the former value being the favourite value in the RMP).

Both RMP and MMP exist in excellent editions. RMP was edited by Peet in 1923 with a hieroglyphic transcription (the original is hieratic), english translation and commentary, and again in 1927-29 by Chace et al, with free translation and commentary (vol. I), reproduction, hieroglyphic transcription, transliteration and literal translation (vol. II). MMP was edited by Struve in 1930 with reproduction, hieroglyphic transcription, German translation and commentary. A new edition of RMP intended for interested laymen has been published in 1987 by Robins & Shute (not yet seen by the present author). A collection of Demotic mathematical papyri was published (with transliteration and English translation and discussion of terminological and technical continuity and change since Middle Kingdom mathematics) by Parker in 1972.

Excellent surveys of Egyptian mathematics have been written by Vogel (1958; in German) and Gillings (1972). The latter work is inspiring but should be used with some caution, since the author often shows what imagined sources *might have looked like*, and does so in the most exquisite and convincing hieratic hand. Both surveys include references to other works and to publications of minor sources. A comprehensive bibliography of works on Egyptian mathematics up to 1929 compiled by Archibald is included in (Chace et al 1927-29); a recent selective bibliography will be found in Dauben (1985: 29-37).

A fictional satirical letter much beloved in the school and reflecting the importance of mathematical computation in scribal occupations has been published by Gardiner (1911).

The »canonical system« of the pictorial arts was described by Iversen (1975). Badawy's exposition of Egyptian architectural design (1965) should be used with circumspection.

The full range of Egyptian mathematics was probably never diffused to the Palestinian area. From the time when the Israelite Kingdoms began approaching a redistributive economy, however, and when the royal scribes came in need of computational tools, epigraphic evidence shows that they took over the Egyptian hieratic numbers (survey of the main evidence in Ifrah 1986: 271). These, however, are more complex than the hieroglyphic numbers which they represent in shorthand, and one can hardly imagine that they were adopted in isolation: They must have been imported together with at least part of that wider mathematical culture which they served. In all probability, the administration in the Divided Kingdom will thus have been effected by means of Egyptian routines and techniques.

F. Greek and Hellenistic mathematics and its aftermath

The third mathematical tradition of some importance in the Biblical context was that of Ancient Greece and of the Hellenistic world.

Early classical Greece was the cradle of »philosophy«, i.e., of intellectual and scientific interest radically separated from direct social utility. While the non-utilitarian stratum of Babylonian (and, as far as it existed, Egyptian) mathematics had to *look like* a tool for scribal practice in order to serve the sociopsychological maintenance of scribal identity, Greek mathematics had to look »pure«, i.e., unbound by social utility. Scribal work, indeed, had become a lowly occupation in Classical Antiquity, and had stopped being intellectually productive.

The starting point was apparently intellectual curiosity *vis-à-vis* the techniques of surveyors and accountants: *Why* did these techniques work? At the end of the road we find Euclid's *Elements*, Archimedes' computations of circle, sphere and paraboloid, and Apollonios' *Conics*, together with a number of minor astronomical works disguised as pure spherical geometry, and Ptolemy's monumental *Almagest*. All of this was fairly irrelevant to both Jewish and Christian culture until the High Middle Ages, and there is no reason to discuss it further here.

More relevant than the majestic cedar forest is the mathematical undergrowth. Several classes of vegetation can be distinguished.

First there is the *alphabetic number system* (see NUMBERS AND COUNTING). It has been much disputed who were the first to use the letters of the alphabet for numbers, and the question is not settled definitively. The Greeks did so at least from the late third century B.C. onwards. So did also the Jews and other Semitic peoples; no evidence for this, however, can be dated before the first century A.D., and until then another system was in use. For this (and various other) reasons it is thus the most reasonable assumption that the alphabetic number system was invented by the Greeks and then taken over by others in the Hellenistic world (see the discussion in Ifrah 1986: 286-302).

Originally, this was just a clever notation for numbers; soon, however, the possibility to read any alphabetic letter as a number was exploited in *gematria*, the substitution of the sum of constituent numbers for a word. An early and most famous example is found in *Apocalypse* 13: 18, the number of the beast being »the number of a man; and the number is 666«. (This reminds of the Assyrian King Sargon's claim concerning the »number of his name«, but the resemblance is probably accidental).

Greater importance was the technique to acquire in Medieval and Renaissance exegesis, *viz.* in the *kabala*, where it was used extensively for symbolic identification of words with the same gematric number (see the description of both Jewish and Christian cabala in Blau 1944).

Next there is the *Pythagorean tradition*. The Pythagorean brotherhood had formed around the late sixth century B.C. around Pythagoras, who was (*pace* an abundance of neo-Pythagorean and modern authors) in all probability no »scientist« or »mathematician« but rather a shamanistic figure, as has been argued by Burkert (1972). Plausibly, however, numerology (on a traditional, »folk« level) was a major ingredient in his doctrine. Over the fifth century, then, and concurrently with the development of scientific mathematics, the Pythagorean brotherhood (or one branch of it) appears to have extended the numerological interest, first by adopting an existing number-theoretical interest (the »doctrine of odd and even«) and extending it, and then by taking up also theoretical geometry. (A satisfactory discussion of the relative chronology of »philosophical« and »Pythagorean« mathematical achievements would lead too far; but see Knorr (1975: *passim*) and Høyrup (1985: 19-21)).

In the fourth century B.C., the Pythagorean movement disappears as a scientific school; throughout Antiquity, however, the basic Pythagorean arithmetical doctrine remains important. It is, in fact, a *doctrine* rather than a theory. The fundamental constituents are the canon of figurate numbers and the classification of numerical proportions. The doctrine was put forth through examples and without proofs. The discoveries made by late Ancient neo-Pythagorean authors (if indeed they made them) were made empirically.

Figurate numbers are the numbers which arise when points are arranged in certain regular patterns. We still know the *square numbers* $1\cdot1$, $2\cdot2$, $3\cdot3$, etc., and the *prime numbers* which can only be arranged in a single row and in no other rectangular pattern. A third species are the *triangular numbers* 1, $1+2$, $1+2+3$, $1+2+3+4$, etc., and still others exist (*rectangular numbers* of the form $n\cdot(n+1)$; *pentagonal numbers* $1+2+...+(n-1)+n^2$; etc.).

The doctrine of proportions was coupled to the theory of musical harmony. An octave corresponds to a ratio 2:1 (in frequency, which the Ancients did not know, and as string lengths on a monochord, which they knew); a fifth corresponds to the ratio 3:2, a fourth to 4:3, etc. All these are *superparticular* ratios, i.e., they have the form $(n+1):n$. Other classes are defined in similar ways.

Neo-Pythagorean arithmetic was considered indispensable for the understanding of (especially Platonic) philosophy, and was hence a prolegomenon in the basic late Ancient philosophical curriculum. In this way, it was spread to much wider circles than high-level mathematics. One place in general culture which was influenced by the Pythagorean doctrine was poetry. A number of texts (so Vergil's *Bucolica* and *Georgica*) are constructed around simple proportions (identity and superparticulars) and prime numbers; these mathematical relations turn up in the counting of lines, words, and letters—especially vowels.

Interestingly, this same technique appears to have been used in the *Gospel According to Luke*. As it is well known, the Sermon on the Mount and the Lord's Prayer are rendered differently by Matthew and Luke; according to the linguist Jens Juhl Jensen (1986), who has compared the Gospel text with the principles used in »Pythagorean« poetry from the same epoch, Luke's version (but not Matthew's) follows Pythagorean principles. It is of some exegetical interest that part of the difference between the two evangelists may be the necessary difference between translation into prose and into poetry governed by strict rules.

Neo-Pythagorean doctrines were also important in Ancient and Medieval exegesis, in particular the figurate numbers. An important character in this connection is Philo of Alexandria, and a good example his discussion of the measures of Noah's Ark (edited by Paramelle (1984: 148-163) with a numerological commentary by Sesiano (205-209)). The length of 300 [cubits] represents the universe, because it is the 24th triangular number, 24 being the number of hours in a day and the number of letters in the Greek alphabet, $24=2^3+2^3+2^3$, and the triad 1+1+1 thus occurring doubly in 24 representing equality (identity of beginning, middle and end), ... Furthermore, $300=(1+3+...+23)+(2+4+...+24)=144+156$, 144 being 12^2 and thus including (as dot patterns) the first 12 squares, while 156= 12·13 »includes« (in the same sense) the first 12 rectangular numbers. So, 300 unites in itself equality and inequality, whereby it is similar to and represents the universe. Similar astute observations are made on the width and the height of the Ark.

Philo's numerology was taken over by both Ambrosius and Augustine (who as a teacher had taught neo-Pythagorean arithmetic in his own youth). But Christian authors until the early Renaissance would also make their own numerological exegesis. A late and beautiful example is Nicholas Cusanus's mathematical »proof« that Trinity could not possibly have been Quaternity (*De docta ignorantia* I, xx; ed. Wilpert 1967 I, 59-60): maximal and minimal entities coincide (a fundamental principle in Cusanus' philosophy); in surveying, the necessary reduction to minimal entities leads to triangulation; *ergo* ... .

»Scientific« Greek mathematics only affected Medieval exegesis on one point. As mentioned above, the »Babylonian π« is accepted in the Bible. This became a problem to Medieval Jewish authors, who devised the explanation that the thread measuring the diameter of the molten sea ran around the inner surface (so explained in *Mišnat ha-Middot* V, 3—ed., tr. Gandz 1932; in fact, the same idea has been proposed not so long ago by the pyramidologist Berriman (1953: 97)).

Summing up the influence of Greek and Hellenistic mathematics we may conlude that it only affected the Biblical text itself in a few and not very central points. As Judaism and (later) Christianity became integrated into general Hellenistic culture, however, neo-Pythagoreanism and elementary Archimedean surveying had become an indisputable (and noncontroversial) part of the intellectual baggage of the Fathers and other commentators, and they would see no problem is using it as a tool for their exegetical efforts. Nor would their disciples in the Middle Ages and the early Renaissance.

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